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Bo Ranneby[1] and Jun Yu

Centre of Biostochastics
Swedish University of Agricultural Sciences, Umeå, Sweden

Abstract

In contingent valuation studies regarding willingness to pay (WTP) the respondents usually give an exact value as his/her WTP-value. Unfortunately, the non-response rate has a tendency to be quite high. As an attempt to reduce that rate the respondents will have a possibility to give a self-selected interval instead of a fixed value as their WTP. In this paper we will study different approaches to estimate the mean willingness to pay under these conditions. First we consider the non-parametric and a parametric approach where the intervals are treated as if the respondent gives an exact value but we cannot observe it. Next we will give a different interpretation of the intervals: Included in the respondent's answer is information about his/her uncertainty about what would be a reasonable value of WTP. For illustration purposes we will use data from a small study in Bollnäs municipality. In all three situations we first estimate the mean WTP and its standard error for those giving a positive answer and finally we add zero-responses.

Keywords: Willingness to pay, cost-benefit analysis, self-selected interval, triangle distribution, self-consistent estimate, contingent valuation.

[1]E-mail address to the correspondence author: bo.ranneby@sekon.slu.se

1 Introduction

The development of methods to measure willingness to pay (WTP) has renewed interest in cost-benefit analysis (CBA) for the economic evaluation of health care programs and environmental issues. We may ask how much people are willing to pay for changes in environmental quality. It depends of course on the individuals' preferences and their income. The preferences are summarized in a utility function u and the willingness to pay is defined as

$$u(y_0, z) = u(y_1, z - WTP). \tag{1}$$

Here y_0 denotes current environmental quality, y_1 improved environmental quality, z income, and WTP the amount the individual is willing to pay for improving environmental quality from y_0 to y_1.

Usually in contingent valuation (CV) studies regarding willingness to pay the respondents give an exact value as his/her WTP-value. Unfortunately, the non-response rate has a tendency to be quite high. As an attempt to reduce that rate the respondents will have a possibility to give a self-selected interval instead of a fixed value as their WTP.

The concept of self-selected intervals is closely related to interval-censored failure time data in survival analysis. Censoring mechanisms can be quite complicated and thus necessitate special methods of treatment. Different types of interval-censored data have been studied. Gehan (1965), Turbull (1974, 1976) and others considered "double-censoring", where an observation gets censored "left and right". Groeneboom and Wellner (1992), Huang (1996) and others studied the Type I interval-censored data (also called as current status data) in which all observed intervals "include" either left- or right-censoring. In CV it means that a single bounded dichotomous choice question records whether a respondent's WTP is either above or below a particular value. Interval-censored data that include at least one finite interval are usually referred to as Type II interval-censored data (Groeneboom and Wellner, 1992; Huang and Wellner, 1997; Sun, 2005; Day, 2007). In CV it means that a respondent's WTP may also be recorded as falling in the interval between two particular values. Readers are referred to Sun (2006) for the data types mentioned above. Huang (1999) and Zhao et al. (2008) considered the partly interval-censored failure time data where observed data include both exact and interval-censored observations on the survival time of interest. Recently Jammalamadaka and Mangalam (2003) introduced the concept of "middle censoring" which occurs when an observation becomes unobservable if it falls inside a random interval.

In this paper we will study three different approaches to estimate the mean willingness to pay under these conditions. First we consider the non-

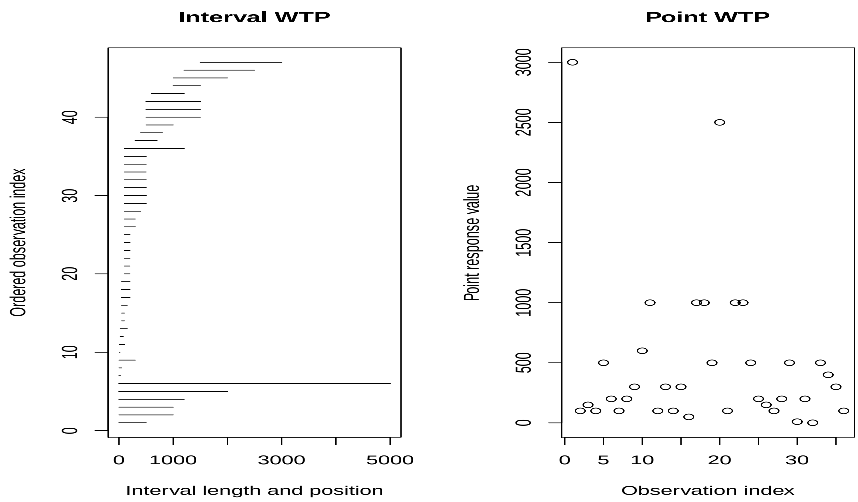

Figure 1: WTP survey data from Bollnäs municipality with 135 responses. Left panel: the 47 interval responses with interval lengths and boundary positions, where the observations are ordered from the lowest left boundary to the highest one; Right panel: the 36 point responses

parametric and a parametric approach where the intervals are treated as if the respondent gives an exact value but we cannot observe it. Next we will give a different interpretation of the intervals: Included in the respondent's answer is information about his/her uncertainty about what would be a reasonable value of WTP. For illustration purposes we will use data from a small study in Bollnäs municipality. In all three situations we first estimate the mean WTP for those giving a positive answer, then the standard error of the estimates, and finally we add zero-responses.

2 Data description

Survey data from Bollnäs municipality on willingness-to-pay for improvements in Bollnäs river are collected, where respondents have the opportunity to give an interval answer. In a web survey with 135 responses, 36 gave point numbers, 47 selected interval-answers, and the rest was zero-responses. The sample size is thus $n = 135$. The data are shown in Figure 1, where the mean of point data is 482.25 and standard deviation is 639.98, and the mean of middle point of intervals is 495.09.

Table 1: The estimated CDF using SCE based on Bollnäs data

Value	1	10	50	100	150	200	300
Prob	0.0132	0.0536	0.1107	0.2881	0.4916	0.601	0.6747
Value	400	500	600	1000	2500	3000	
Prob	0.6946	0.7762	0.8238	0.9268	0.9878	1	

3 WTP estimation

3.1 Non-parametric

First we will treat the non-parametric situation. Assume that the willingness to pay X follows a distribution given by the distribution function F, which takes values in the interval $(0, \infty)$. After rearranging the data (if necessary) such that the first n_1 are exact observations (X_i) and the rest n_2 are intervals (L_i, R_i) with $n_1 + n_2 = n$, we have the following observed data:

$$X_1, X_2, \ldots, X_{n_1}, (L_{n_1+1}, R_{n_1+1}), (L_{n_1+2}, R_{n_1+2}), \ldots, (L_{n_1+n_2}, R_{n_1+n_2}). \quad (2)$$

The likelihood $L(F)$ can be written as follows:

$$L(F) = \prod_{i=1}^{n_1} [F(X_i) - F(X_i-)] \prod_{i=n_1+1}^{n} [F(R_i) - F(L_i)]. \quad (3)$$

The nonparametric maximum likelihood estimate (NPMLE) $\hat{F}_n$ is the maximizer of $L(F)$ in the class of distribution functions on the line.

This situation with partially observed values in a non-parametric context has been studied among others by Turnbull (1976) and by Jamalamadaka and Mangalam (2003). Although the assumptions generating the intervals are slightly different the likelihood function will be the same. We will follow the approach by Jamalamadaka and Mangalam (2003) and obtain a self-consistent estimate (SCE) of the distribution function. The estimated cumulative distribution function (CDF) for the Bollnäs study is presented in Table 1, where "Value" is the observed exact WTP value or interval boundaries, and "Prob" is the probability $\mathbb{P}(X \leq Value)$.

Figure 2 shows the SCE and the empirical CDF (from R function "ecdf") only for the point observations. It seems that the SCE usually gives greater probabilities than the ECDF at the same points.

The expected willingness to pay is given by

$$\mathbb{E}(X) = \int [1 - F(x)] dx.$$

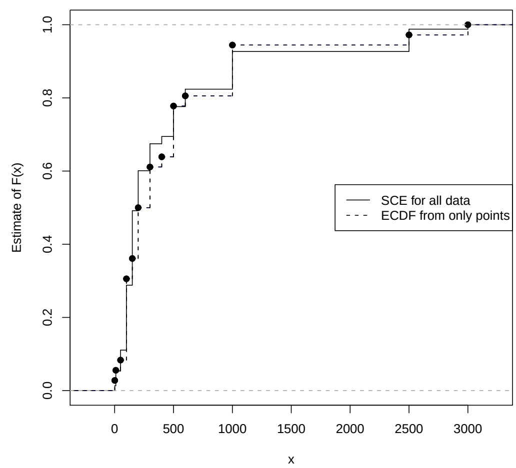

Figure 2: Estimation of $F(x)$ by using the SCE and ECDF (for only point WTP)

Jammalamadaka and Mangalam (2003) established the strong consistency of $\hat{F}_n$ under some regularity conditions, i.e.

$$\sup_{x>0} |\hat{F}_n(x) - F(x)| \to 0$$

with probability one as $n \to \infty$. Thus it is natural to estimate the mean WTP by

$$\widehat{\mathbb{E}}(X) = \int [1 - \hat{F}_n(x)]dx.$$

In the same way the second order moments can be estimated by

$$\widehat{\mathbb{E}}(X^2) = \int [1 - \hat{F}_n(\sqrt{x})]dx.$$

To ensure that the first and second order moments converge to the right value it requires that $\hat{F}_n$ is uniformly integrable, which is always satisfied if the support is finite.

3.2 Parametric (Weibull)

Next the parametric approach is treated. The Weibull distribution is chosen because of its flexibility. It can mimic the behavior of other statistical distributions such as the normal and the exponential. The density f of Weibull

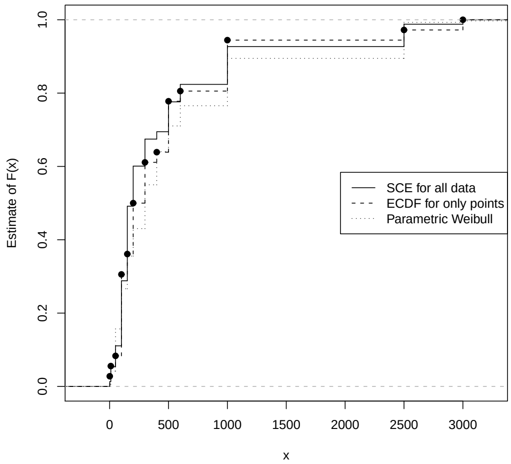

Figure 3: Estimation of $F(x)$ by using the SCE, ECDF, and Weibull probabilities

distribution F with shape parameter $\kappa > 0$ and scale parameter $\lambda > 0$ is given by

$$f(x; \kappa, \lambda) = \frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{(\kappa-1)} e^{-(x/\lambda)^{\kappa}}, \qquad \text{for } x \geq 0.$$

The mean is $\lambda\Gamma(1 + \frac{1}{\kappa})$ and the variance is $\lambda^2[\Gamma(1 + \frac{2}{\kappa}) - \Gamma^2(1 + \frac{1}{\kappa})]$.

When the data as in (2) are observed, the likelihood function $L(X; \kappa, \lambda)$ can be written as follows

$$L(X; \kappa, \lambda) = \prod_{i=1}^{n_1} f(X_i; \kappa, \lambda) \prod_{i=n_1+1}^{n_1+n_2} [F(R_i; \kappa, \lambda) - F(L_i; \kappa, \lambda)]. \tag{4}$$

Using the maximum likelihood estimator (MLE) from R STAR package and others as starting values, the parameter (κ, λ) estimation always converges to about (0.8613, 389.72). The probabilities of this Weibull distribution at the same points are shown in Figure 3, together with the SCE and ECDF. It is seen that Weibull usually gives smaller probabilities than the other two densities at larger values of WTP.

3.3 Interval responses as a measure of uncertainty

In this section we will give a different interpretation of interval answers. We assume that, because of a number of uncertainties, instead of giving an exact value the answer is given by a random variable having a certain distribution,

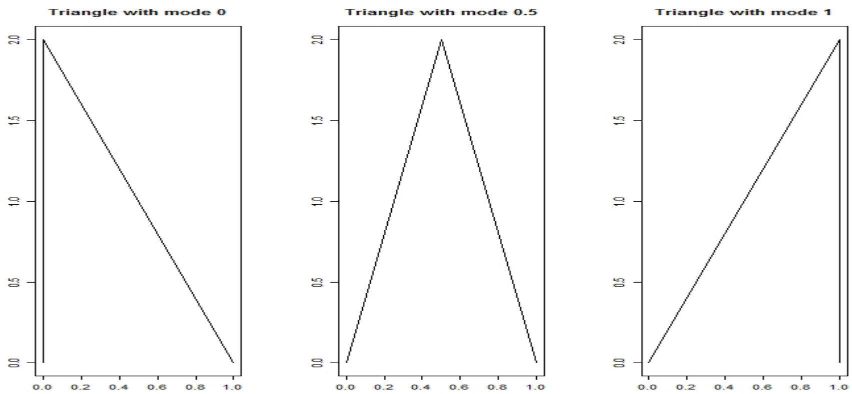

Figure 4: An example of triangular density with different modes

i.e. the WTP in equation (1) for individual i is given by W_i having mean value X_i. (Of course the expected value is a constant but randomness occurs because of the selection of individuals to the panel). Certainly the respondent cannot give the answer as a distribution but has to approximate it by giving the lower and upper value of the interval and it is default what kind of distribution the respondent has in mind. The natural choices are a uniform distribution or a triangular distribution. Although symmetric distributions are most natural we will also consider triangular distributions with mode in the left or right end of the interval. See Figure 4 for an example of triangular densities with different modes.

For an interval observation $[L_i, R_i]$, the triangular distributions with L_i as the lower limit, R_i as the upper limit, and modes at L_i, $\frac{L_i+R_i}{2}$ and R_i (denoted as mode at 0, 0.5, and 1 in Tables 2-4) respectively, or a uniform distribution are analyzed.

Suppose that the WTP for the ith respondent is W_i. After rearranging the data as in (2), we have the following observed data:

$$W_i = \begin{cases} X_i, & \text{for exact responses } i = 1, \dots, n_1 \\ X_i + \varepsilon_i, & \text{for interval responses } i = n_1 + 1, \dots, n_1 + n_2 (= n) \end{cases} \quad (5)$$

where X_i is the “true" or expected WTP for the respondent and ε_i is a random variable, independent of X_i, that indicates the uncertainty in the answer.

As mentioned before, we assume that ε_i has a triangular distribution (with different modes) or a uniform distribution. Intuitively, the wider an interval one answers, the greater uncertainty a respondent has. In fact, for an interval

Table 2: The estimates of mean WTP and their standard errors by the three different methods without Zero-responses: nonparametric (SCE), parametric (Weibull), and our method

	SCE	Weibull	Triangular dist. with mode at 0	0.5	1	Uniform distribution
WTP	464.9	420.6	436.5	489.5	542.5	489.5
s.e.	72.48	53.79	63.32	68.08	75.73	70.03

observation $[L_i, R_i]$, the variances for ε_i are $\frac{(R_i-L_i)^2}{18}$, $\frac{(R_i-L_i)^2}{24}$, $\frac{(R_i-L_i)^2}{18}$, and $\frac{(R_i-L_i)^2}{12}$ for a triangular distribution with mode at 0, 0.5, and 1,or a uniform distribution, respectively.

Taking the interval uncertainty into account, the variance of the total WTP is

$$Var\left[\sum_{i=1}^{n} W_i\right] = Var\left[\sum_{i=1}^{n} X_i\right] + Var\left[\sum_{i=n_1+1}^{n} \varepsilon_i\right] = nVar(X) + \sum_{i=n_1+1}^{n} \frac{(R_i - L_i)^2}{c}, \tag{6}$$

where

$$c = \begin{cases} 12, & \text{for uniform distribution;} \\ 18, & \text{for triangular distribution with mode at 0 or 1;} \\ 24, & \text{for triangular distribution with mode at 0.5.} \end{cases}$$

From (6) we can estimate the variance of mean WTP, $Var(\bar{W})$, by

$$\widehat{Var}(\overline{W}) = \frac{S_X^2}{n} + \frac{1}{n^2} \sum_{i=n_1+1}^{n} \frac{(R_i - L_i)^2}{c}, \tag{7}$$

where S_X^2 is the sample variance of $\{X_i\}$.

Table 2 presents the estimation results for mean WTP and it standard error, using the three different methods: the nonparametric one based on SCE, parametric based on Weibull distribution, and our method based on uncertainty measure. We observe that the Weibull method has the lowest standard error but the estimated WTP is also smallest, even less than the one that assuming all the interval answers having the mode at the lower limits. So the WTP seems to be underestimated. Whereas the WTP estimates with SCE, the triangular distribution with mode at middle, and the uniform distribution look reasonable, the triangular is favorable with its lower standard error.

Table 3: The estimates of the standard errors for the two extreme cases

	Triangular dist. with mode at 0	0.5	1	Uniform distribution
s.e.(I)	60.41	66.07	73.32	66.07
s.e.(II)	65.45	69.58	77.52	72.9

Table 4: The estimates of mean WTP and their standard errors by the three different methods with Zero-responses

	SCE	Weibull	Triangular dist. with mode at 0	0.5	1	Uniform distribution
WTP	285.8	258.6	268.4	301.0	333.5	301.0
s.e.	44.56	33.07	37.14	40.62	45.08	40.62

To investigate the behavior of uncertainty in our WTP estimation, it would be interesting to know how the standard error changes in the following two extreme cases:

(I) The interval answers were replaced by their middle point so that we pretend that we had only observed exact points.

(II) Opposite to Case I, we pretend that we had only observed intervals.

The standard errors estimated corresponding to these two cases are shown in Table 3. The changing rate seems quite small, which implied that our estimate of uncertainty is stable.

Table 4 presents similar information as Table 2, but with zero-responses included. The results in Table 4 are easily obtained by multiplying the corresponding information in Table 2 by a factor 83/135 (total number of non-zero responses/total number of responses).

4 Discussion

Contingent valuation surveys frequently employ elicitation procedures that return interval-censored data on respondent's willingness to pay. In this paper we introduce a new interpretation of CV interval-censored responses: point and self-selected interval responses, which differs from Type II interval-censoring or Middle censoring. A new model based on this interpretation is proposed. By using CV survey data from a small study in Bollnäs municipality, the mean WTP and its standard error were estimated and then compared with a nonparametric approach based on SCE for middle censoring and a parametric approach based on Weibull distribution. Summarizing the results, one can conclude that the parametric approach underestimates the WTP and the SCE approach tends to underestimate the WTP and has also larger relative standard error, compared to our method.

A closer look at Table 2 shows that the relative standard errors varies from 12.8% for the Weibull approach to 15.6% for the non-parametric approach (SCE). The Weibull approach has the lowest relative standard error but at the same time the estimate of mean willingness to pay is remarkably smaller compared to the other methods. In fact it is even smaller than when we assume that the individual's uncertainty is given by triangular distribution with mode at the left end of the reported interval. Thus there is an evident risk that the Weibull distribution does not describe the data sufficiently well and leads to biased estimates. Assuming a parametric model gives more structure to the problem and often also to lower standard errors of the estimates. Unfortunately if there is no underlying knowledge about the chosen parametric model but it is chosen by other reasons e.g. flexibility the risk for biased estimates is obvious.

The mean WTP estimate using the SCE approach (464.9) is lower than the one based on symmetric distributions (triangular and uniform). It is also worth noting that it is clearly lower than the sample mean WTP obtained from those exact responses, which is 482.3. This indicates that the SCE approach tend to underestimate the mean WTP. However, it's not clear for us whether this is a drawback in methodology itself.

The relative standard errors when we assume different triangular distributions are all around 14%, which is lower than that obtained from the SCE approach. It is interesting to compare the situation where it is assumed that all respondents give an exact value or all give an interval. When interval uncertainty is attributed to all respondents the standard errors increases form 5% (symmetric triangular distribution) to 10% (uniform distribution) or with another interpretation. To obtain the same precision the number of respondents has to increase with 10% (symmetric triangular distribution) to 21% (uniform distribution). One of the reasons to allow self-selected intervals was to reduce the non-response rate. Usually, the non-respondents have other preferences so to avoid bias a second small sample from the non-respondents should be included in the study. Thus by increasing the sample size slightly and allow self-selected intervals we guarantee a higher precision and may also avoid the need of a second sample.

Acknowledgement

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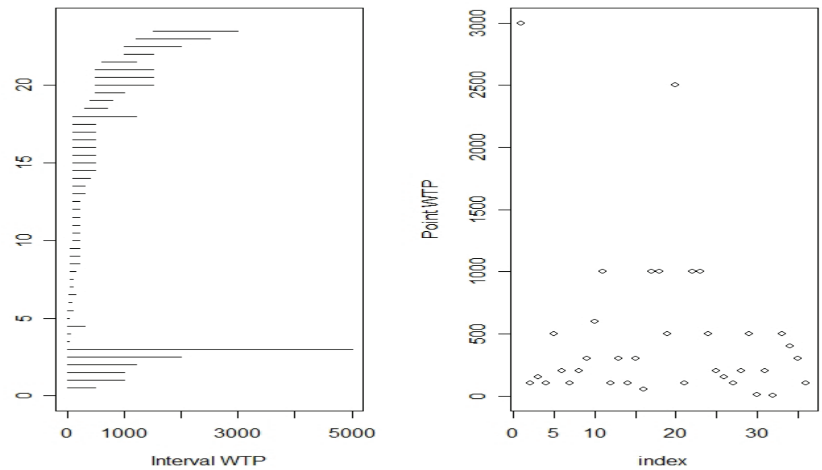

Figure 1: The interval and point data for WTP

In this paper we will study three different approaches to estimate the mean willingness to pay under these conditions. First we consider the non-parametric and a parametric approach where the intervals are treated as if the respondent gives an exact value but we cannot observe it. Next we will give a different interpretation of the intervals: Included in the respondent's answer is information about his/her uncertainty about what would be a reasonable value of WTP. For illustration purposes we will use data from a small study in Bollnäs municipality. In all three situations we first estimate the mean WTP for those giving a positive answer, then the standard error of the estimates, and finally we add zero-responses.

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3 WTP estimation

3.1 Non-parametric

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The likelihood $L(F)$ can be written as follows:

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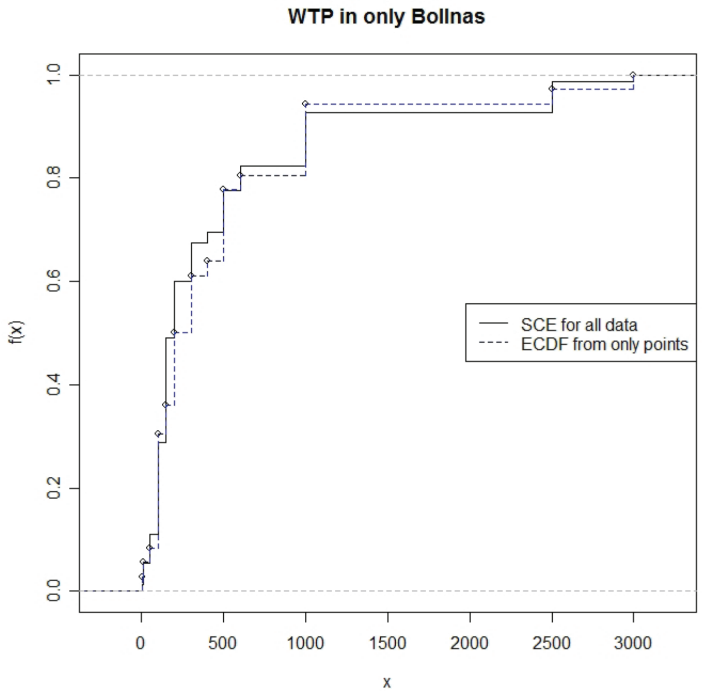
$$\mathbb{E}(X) = \int [1 - F(x)] dx.$$



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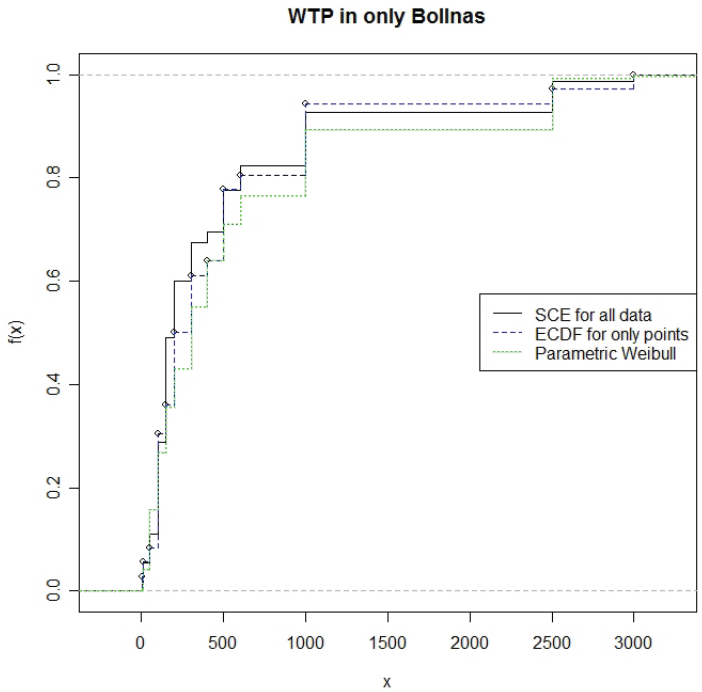

Figure 3: The SCE, ECDF and Weibull probabilities

3.2 Parametric (Weibull)

Next the parametric approach is treated. The Weibull distribution is chosen because of its flexibility. It can mimic the behavior of other statistical distributions such as the normal and the exponential. The density f of Weibull distribution F with shape parameter $\kappa > 0$ and scale parameter $\lambda > 0$ is given by

$$f(x;\kappa,\lambda) = \frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{(\kappa-1)} e^{-(x/\lambda)^{\kappa}}, \qquad \text{for } x \geq 0.$$

The mean is $\lambda\Gamma(1+\frac{1}{\kappa})$ and the variance is $\lambda^2[\Gamma(1+\frac{2}{\kappa})-\Gamma^2(1+\frac{1}{\kappa})]$.

When the data as in (2) are observed, the likelihood function $L(X;\kappa,\lambda)$ can be written as follows

$$L(X;\kappa,\lambda) = \prod_{i=1}^{n_1} f(X_i;\kappa,\lambda) \prod_{i=n_1+1}^{n_1+n_2} [F(R_i;\kappa,\lambda) - F(L_i;\kappa,\lambda)]. \tag{4}$$

Using the maximum likelihood estimator (MLE) from R STAR package and others as starting values, the parameter (κ,λ) estimation always converges

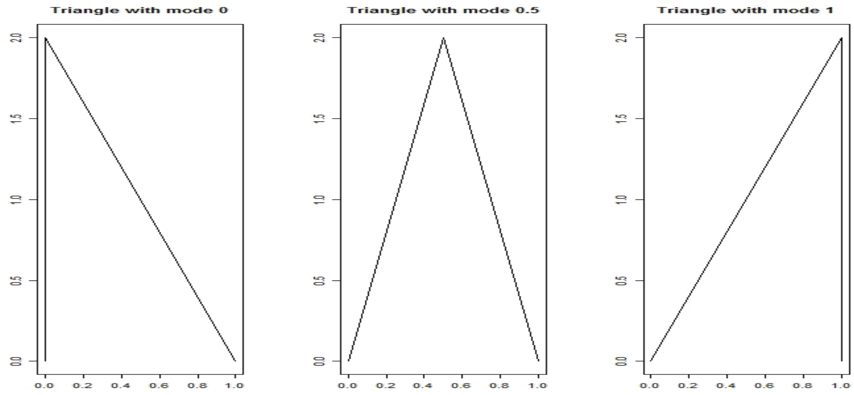

Figure 4: An example of triangular density with different modes

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In this section we will give a different interpretation of interval answers. We assume that, because of a number of uncertainties, instead of giving an exact value the answer is given by a random variable having a certain distribution, i.e. the WTP in equation (1) for individual i is given by W_i having mean value X_i. (Of course the expected value is a constant but randomness occurs because of the selection of individuals to the panel). Certainly the respondent cannot give the answer as a distribution but has to approximate it by giving the lower and upper value of the interval and it is default what kind of distribution the respondent has in mind. The natural choices are a uniform distribution or a triangular distribution. Although symmetric distributions are most natural we will also consider triangular distributions with mode in the left or right end of the interval. See Figure 4 for an example of triangular densities with different modes.

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Suppose that the WTP for the ith respondent is W_i. After rearranging the data as in (2), we have the following observed data:

$$W_i = \begin{cases} X_i, & \text{for exact responses } i = 1, \ldots, n_1 \\ X_i + \varepsilon_i, & \text{for interval responses } i = n_1 + 1, \ldots, n_1 + n_2 (= n) \end{cases} \tag{5}$$

where X_i is the "true" or expected WTP for the respondent and ε_i is a random variable, independent of X_i, that indicates the uncertainty in the answer.

As mentioned before, we assume that ε_i has a triangular distribution (with different modes) or a uniform distribution. Intuitively, the wider an interval one answers, the greater uncertainty a respondent has. In fact, for an interval observation $[L_i, R_i]$, the variances for ε_i are $\frac{(R_i-L_i)^2}{18}$, $\frac{(R_i-L_i)^2}{24}$, $\frac{(R_i-L_i)^2}{18}$, and $\frac{(R_i-L_i)^2}{12}$ for a triangular distribution with mode at 0, 0.5, and 1,or a uniform distribution, respectively.

Taking the interval uncertainty into account, the variance of the total WTP is

$$Var\left[\sum_{i=1}^{n} W_i\right] = Var\left[\sum_{i=1}^{n} X_i\right] + Var\left[\sum_{i=n_1+1}^{n} \varepsilon_i\right] = nVar(X) + \sum_{i=n_1+1}^{n} \frac{(R_i - L_i)^2}{c}, \tag{6}$$

where

$$c = \begin{cases} 12, & \text{for uniform distribution;} \\ 18, & \text{for triangular distribution with mode at 0 or 1;} \\ 24, & \text{for triangular distribution with mode at 0.5.} \end{cases}$$

From (6) we can estimate the variance of mean WTP, $Var(\bar{W})$, by

$$\widehat{Var}(\overline{W}) = \frac{S_X^2}{n} + \frac{1}{n^2} \sum_{i=n_1+1}^{n} \frac{(R_i - L_i)^2}{c}, \tag{7}$$

where S_X^2 is the sample variance of $\{X_i\}$.

Table 2 presents the estimation results for mean WTP and it standard error, using the three different methods: the nonparametric one based on SCE, parametric based on Weibull distribution, and our method based on uncertainty measure. We observe that the Weibull method has the lowest standard error but the estimated WTP is also smallest, even less than the one that assuming all the interval answers having the mode at the lower limits. So the WTP seems to be underestimated. Whereas the WTP estimates with SCE, the triangular distribution with mode at middle, and the uniform distribution look reasonable, the triangular is favorable with its lower standard error.

Table 2: The estimates of mean WTP and their standard errors by the three different methods without Zero-responses: nonparametric (SCE), parametric (Weibull), and our method

	SCE	Weibull	Triangular dist. with mode at 0	0.5	1	Uniform distribution
WTP	464.9	420.6	436.5	489.5	542.5	489.5
s.e.	72.48	53.79	63.32	68.08	75.73	70.03

Table 3: The estimates of the standard errors for the two extreme cases

	Triangular dist. with mode at 0	0.5	1	Uniform distribution
s.e.(I)	60.41	66.07	73.32	66.07
s.e.(II)	65.45	69.58	77.52	72.9

To investigate the behavior of uncertainty in our WTP estimation, it would be interesting to know how the standard error changes in the following two extreme cases:

(I) The interval answers were replaced by their middle point so that we pretend that we had only observed exact points.

(II) Opposite to Case I, we pretend that we had only observed intervals.

The standard errors estimated corresponding to these two cases are shown in Table 3. The changing rate seems quite small, which implied that our estimate of uncertainty is stable.

Table 4 presents similar information as Table 2, but with zero-responses included. The results in Table 4 are easily obtained by multiplying the corresponding information in Table 2 by a factor 83/135 (total number of non-zero responses/total number of responses).

4 Discussion

Contingent valuation surveys frequently employ elicitation procedures that return interval-censored data on respondent's willingness to pay. In this paper we introduce a new interpretation of CV interval-censored responses:

Table 4: The estimates of mean WTP and their standard errors by the three different methods with Zero-responses

	SCE	Weibull	Triangular dist. with mode at 0	0.5	1	Uniform distribution
WTP	285.8	258.6	268.4	301.0	333.5	301.0
s.e.	44.56	33.07	37.14	40.62	45.08	40.62

point and self-selected interval responses, which differs from Type II interval-censoring or Middle censoring. A new model based on this interpretation is proposed. By using CV survey data from a small study in Bollnäs municipality, the mean WTP and its standard error were estimated and then compared with a nonparametric approach based on SCE for middle censoring and a parametric approach based on Weibull distribution. Summarizing the results, one can conclude that the parametric approach underestimates the WTP and the SCE approach tends to underestimate the WTP and has also larger relative standard error, compared to our method.

A closer look at Table 2 shows that the relative standard errors varies from 12.8% for the Weibull approach to 15.6% for the non-parametric approach (SCE). The Weibull approach has the lowest relative standard error but at the same time the estimate of mean willingness to pay is remarkably smaller compared to the other methods. In fact it is even smaller than when we assume that the individual's uncertainty is given by triangular distribution with mode at the left end of the reported interval. Thus there is an evident risk that the Weibull distribution does not describe the data sufficiently well and leads to biased estimates. Assuming a parametric model gives more structure to the problem and often also to lower standard errors of the estimates. Unfortunately if there is no underlying knowledge about the chosen parametric model but it is chosen by other reasons e.g. flexibility the risk for biased estimates is obvious.

The mean WTP estimate using the SCE approach (464.9) is lower than the one based on symmetric distributions (triangular and uniform). It is also worth noting that it is clearly lower than the sample mean WTP obtained from those exact responses, which is 482.3. This indicates that the SCE approach tend to underestimate the mean WTP. However, it's not clear for us whether this is a drawback in methodology itself.

The relative standard errors when we assume different triangular distributions are all around 14%, which is lower than that obtained from the SCE approach. It is interesting to compare the situation where it is assumed that all respondents give an exact value or all give an interval. When interval uncertainty is included to all respondents the standard errors increases form 5% (symmetric triangular distribution) to 10% (uniform distribution) or with another interpretation. To obtain the same precision the number of respondents has to increase with 10% (symmetric triangular distribution) to 21% (uniform distribution). One of the reasons to allow self-selected intervals was to reduce the non-response rate. Usually, the non-respondents have other preferences so to avoid bias a second small sample from the non-respondents should be included in the study. Thus by increasing the sample size slightly and allow self-selected intervals we guarantee a higher precision and may also avoid the need of a second sample.

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