# The first example

The dataset is a 2x2 factorial design with two replicates:



Read the dataset into R and call it a:

a <- data.frame(A = as.factor(rep(c(1,1,1,1,2,2,2,2))),

B = as.factor(rep(c(1,1,2,2,1,1,2,2))),

y = c(6.0,5.5,6.2,5.0,1.2,1.1,0.9,0.4))

Do the “default” analysis and use anova to get the table with Type I tests

mod.def <- lm(y ~ A \* B, data=a) # mod.def means model.default

anova(mod.def) # anova uses Type I tests

Use the library car that by default uses Type II tests:

car::Anova(mod.def) # Anova in car uses Type II tests

Use the library car but choose the Type III tests:

car::Anova(mod.def, type = "III")

Look at the design matrix of the model:

(mm.def <- model.matrix(mod.def))

Remove the column for factor A in the model matrix:

mod.def\_noA <- lm(y ~ mm.def[,-2],data=a)

Look at the difference in residuals to confirm that this is the method used for Type III in Anova:

sum(residuals(mod.def\_noA)^2) - sum(residuals(mod.def)^2)

Redefine the parametrization as is standard, the sum of the parameters is 0:

contrasts(a$A) <- contr.sum

contrasts(a$B) <- contr.sum

contrasts(a$A)

contrasts(a$B)

Redo the calculation with these contrasts to confirm that this is Type II and Type III in SAS

mod.sum <- lm(y ~ A \* B, data=a)

car::Anova(mod.sum, type = "III")

(mm.sum <- model.matrix(mod.sum))

sum(residuals(mod.sum\_noA)^2) - sum(residuals(mod.sum)^2)

Another way to the correct solution

contrasts(a$A) <- contr.treatment

contrasts(a$B) <- contr.treatment

A\_B <- with(a, (A==B))

mod.alt <- lm(y ~ 1 + A + B + A\_B, data=a)

(mm.alt <- model.matrix(mod.alt))

car::Anova(mod.alt, type = "III")

mod.alt\_noA <- lm(y ~ mm.alt[,-2],data=a)

sum(residuals(mod.alt\_noA)^2) - sum(residuals(mod.alt)^2)

Program and output from SAS:

**data** a;

input A B y;

datalines;

1 1 6

1 1 5.5

1 2 6.2

1 2 5

2 1 1.2

2 1 1.1

2 2 0.9

2 2 0.4

run;

**proc** **mixed** data=a;

title 'With type III';

class A B;

model y=A B A\*B;

**run**;

With type III

The Mixed Procedure

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

A 1 4 187.08 0.0002

B 1 4 0.87 0.4046

A\*B 1 4 0.25 0.6425

# Split-plot analysis

|  |  |  |  |
| --- | --- | --- | --- |
| Replicate | Variety | NitroF | Yield |
| I | Victory | 0.0cwt | 111 |
| I | Victory | 0.2cwt | 130 |
| I | Victory | 0.4cwt | 157 |
| I | Victory | 0.6cwt | 174 |
| I | Golden.rain | 0.0cwt | 117 |
| I | Golden.rain | 0.2cwt | 114 |
| I | Golden.rain | 0.4cwt | 161 |
| I | Golden.rain | 0.6cwt | 141 |
| I | Marvellous | 0.0cwt | 105 |
| I | Marvellous | 0.2cwt | 140 |
| I | Marvellous | 0.4cwt | 118 |
| I | Marvellous | 0.6cwt | 156 |
| II | Victory | 0.0cwt | 61 |
| II | Victory | 0.2cwt | 91 |
| II | Victory | 0.4cwt | 97 |
| II | Victory | 0.6cwt | 100 |
| II | Golden.rain | 0.0cwt | 70 |
| II | Golden.rain | 0.2cwt | 108 |
| II | Golden.rain | 0.4cwt | 126 |
| II | Golden.rain | 0.6cwt | 149 |
| II | Marvellous | 0.0cwt | 96 |
| II | Marvellous | 0.2cwt | 124 |
| II | Marvellous | 0.4cwt | 121 |
| II | Marvellous | 0.6cwt | 144 |
| III | Victory | 0.0cwt | 68 |
| III | Victory | 0.2cwt | 64 |
| III | Victory | 0.4cwt | 112 |
| III | Victory | 0.6cwt | 86 |
| III | Golden.rain | 0.0cwt | 60 |
| III | Golden.rain | 0.2cwt | 102 |
| III | Golden.rain | 0.4cwt | 89 |
| III | Golden.rain | 0.6cwt | 96 |
| III | Marvellous | 0.0cwt | 89 |
| III | Marvellous | 0.2cwt | 129 |
| III | Marvellous | 0.4cwt | 132 |
| III | Marvellous | 0.6cwt | 124 |
| IV | Victory | 0.0cwt | 74 |
| IV | Victory | 0.2cwt | 89 |
| IV | Victory | 0.4cwt | 81 |
| IV | Victory | 0.6cwt | 122 |
| IV | Golden.rain | 0.0cwt | 64 |
| IV | Golden.rain | 0.2cwt | 103 |
| IV | Golden.rain | 0.4cwt | 132 |
| IV | Golden.rain | 0.6cwt | 133 |
| IV | Marvellous | 0.0cwt | 70 |
| IV | Marvellous | 0.2cwt | 89 |
| IV | Marvellous | 0.4cwt | 104 |
| IV | Marvellous | 0.6cwt | 117 |
| V | Victory | 0.0cwt | 62 |
| V | Victory | 0.2cwt | 90 |
| V | Victory | 0.4cwt | 100 |
| V | Victory | 0.6cwt | 116 |
| V | Golden.rain | 0.0cwt | 80 |
| V | Golden.rain | 0.2cwt | 82 |
| V | Golden.rain | 0.4cwt | 94 |
| V | Golden.rain | 0.6cwt | 126 |
| V | Marvellous | 0.0cwt | 63 |
| V | Marvellous | 0.2cwt | 70 |
| V | Marvellous | 0.4cwt | 109 |
| V | Marvellous | 0.6cwt | 99 |
| VI | Victory | 0.0cwt | 53 |
| VI | Victory | 0.2cwt | 74 |
| VI | Victory | 0.4cwt | 118 |
| VI | Victory | 0.6cwt | 113 |
| VI | Golden.rain | 0.0cwt | 89 |
| VI | Golden.rain | 0.2cwt | 82 |
| VI | Golden.rain | 0.4cwt | 86 |
| VI | Golden.rain | 0.6cwt | 104 |
| VI | Marvellous | 0.0cwt | 97 |
| VI | Marvellous | 0.2cwt | 99 |
| VI | Marvellous | 0.4cwt | 119 |
| VI | Marvellous | 0.6cwt | 121 |

For the R-program and explanations, see the script.

Program and output from SAS:

**proc** **mixed** data=splitplot;

title 'Splitplot in SAS';

class Replicate Variety NitroF;

model Yield=Variety NitroF Variety\*NitroF / ddfm=kr;

random Replicate Replicate\*Variety;

**run**;

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

Variety 2 10 1.49 0.2724

NitroF 3 45 37.69 <.0001

Variety\*NitroF 6 45 0.30 0.9322

# Where are the dfs in ML?

Farm Size y

1 1 49

1 2 63

1 3 58

1 4 59

1 5 53

2 1 55

2 2 62

2 3 62

2 4 63

2 5 74

3 1 55

3 2 57

3 3 61

3 4 62

3 5 55

4 1 49

4 2 55

4 3 48

4 4 57

4 5 56

5 1 59

5 2 57

5 3 60

5 4 55

5 5 54

6 1 49

6 2 53

6 3 60

6 4 52

6 5 58

7 1 60

7 2 62

7 3 60

7 4 53

7 5 59

8 1 50

8 2 54

8 3 68

8 4 58

8 5 49

For the R-program and explanations, see the script.

Program and output from SAS. Note that the SAS-program with ML and Kenward-Roger has 32 df but in R it is 28 df.

**proc** **mixed** data=hens;

title 'With method=REML (default) and Satterthwite';

class farm size;

model y=size / ddfm=satterth;

random farm;

**run**;

With method=REML (default) and Satterthwite

The Mixed Procedure

Covariance Parameter

Estimates

Cov Parm Estimate

Farm 5.0589

Residual 22.7018

Fit Statistics

-2 Res Log Likelihood 224.2

AIC (smaller is better) 228.2

AICC (smaller is better) 228.6

BIC (smaller is better) 228.4

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

Size 4 28 1.93 0.1333

**proc** **mixed** data=hens method=ML;

title 'With method=ML and Satterthwite';

class farm size;

model y=size / ddfm=satterth;

random farm;

**run**;

With method=ML and Satterthwite

The Mixed Procedure

Covariance Parameter

Estimates

Cov Parm Estimate

Farm 4.4266

Residual 19.8641

Fit Statistics

-2 Log Likelihood 239.1

AIC (smaller is better) 253.1

AICC (smaller is better) 256.6

BIC (smaller is better) 253.6

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

Size 4 32 2.20 0.0908

**proc** **mixed** data=hens;

title 'With method=REML (default) and Kenward-Rogers';

class farm size;

model y=size / ddfm=kr;

random farm;

**run**;

With method=REML (default) and Kenward-Rogers 27

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The Mixed Procedure

Covariance Parameter

Estimates

Cov Parm Estimate

Farm 5.0589

Residual 22.7018

Fit Statistics

-2 Res Log Likelihood 224.2

AIC (smaller is better) 228.2

AICC (smaller is better) 228.6

BIC (smaller is better) 228.4

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

Size 4 28 1.93 0.1333

**proc** **mixed** data=hens method=ML;

title 'With method=ML and Kenward-Rogers';

class farm size;

model y=size / ddfm=kr;

random farm;

**run**;

With method=ML and Kenward-Rogers

The Mixed Procedure

Covariance Parameter

Estimates

Cov Parm Estimate

Farm 4.4266

Residual 19.8641

Fit Statistics

-2 Log Likelihood 239.1

AIC (smaller is better) 253.1

AICC (smaller is better) 256.6

BIC (smaller is better) 253.6

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

Size 4 32 2.20 0.0908

# Amusing example



**data** four;

input A B y;

datalines;

0 0 0.26961

0 0 -0.62999

0 0 0.86866

0 0 1.72720

0 0 0.02419

0 1 1.36803

0 1 -0.30920

0 1 1.73862

0 1 1.04487

0 1 -0.04840

1 0 1.72785

1 0 -1.17860

1 0 0.65321

1 0 -0.36857

1 0 -0.59955

1 0 0.05461

1 0 1.70768

1 0 -1.09437

1 0 -0.28928

1 0 2.20741

1 0 0.51875

1 0 -1.40492

1 0 2.01486

1 0 -1.18816

1 0 0.19038

1 0 -1.16974

1 0 -0.03808

1 0 2.35420

1 0 1.39343

1 0 -0.56033

1 0 -0.67146

1 0 0.49244

1 0 -1.17939

1 0 -1.05872

1 0 1.13790

1 0 -0.16027

1 0 0.63049

1 0 1.61696

1 0 -0.19350

1 0 -1.60779

1 0 -0.88516

1 0 -0.43233

1 0 -0.42162

1 0 -0.17049

1 0 0.24581

1 0 -0.74575

1 0 -0.27394

1 0 1.82458

1 0 0.01423

1 0 0.18804

1 0 -0.05413

1 0 0.46161

1 0 -0.59677

1 0 1.26326

1 0 -1.14533

1 0 1.08462

1 0 -1.52900

1 0 -1.57374

1 0 -0.11409

1 0 0.11121

1 1 1.21414

1 1 1.55378

1 1 -0.05952

1 1 -0.61005

1 1 0.66185

1 1 1.20494

1 1 0.77565

1 1 0.09039

1 1 0.19189

1 1 1.55308

1 1 0.61082

1 1 0.55275

1 1 0.97886

1 1 0.40058

1 1 0.68913

1 1 0.31837

1 1 0.79794

1 1 2.11680

1 1 1.82600

1 1 2.25092

1 1 3.60810

1 1 0.94896

1 1 3.22719

1 1 0.98613

1 1 -0.54740

1 1 -0.37989

1 1 2.47987

1 1 0.74508

1 1 0.67372

1 1 0.27336

1 1 -0.95235

1 1 1.42294

1 1 2.18168

1 1 1.91796

1 1 1.09547

1 1 -0.68443

1 1 1.99033

1 1 0.29217

1 1 0.40597

1 1 -0.06590

1 1 0.05923

1 1 1.09391

1 1 1.06625

1 1 0.00934

1 1 0.92789

1 1 0.77674

1 1 0.85722

1 1 1.53541

1 1 0.62544

1 1 0.08184

**run**;

**proc** **mixed** data=four;

title 'With type III';

class A B;

model y=A B A\*B / E solution ddfm=kr;

**run**;

Type 2 Coefficients for B

Effect A B Row1

Intercept

A 0

A 1

B 0 1

B 1 -1

A\*B 0 0 0.0909

A\*B 0 1 -0.091

A\*B 1 0 0.9091

A\*B 1 1 -0.909

Type 3 Coefficients for B

Effect A B Row1

Intercept

A 0

A 1

B 0 1

B 1 -1

A\*B 0 0 0.5

A\*B 0 1 -0.5

A\*B 1 0 0.5

A\*B 1 1 -0.5

The Mixed Procedure

Type 3 Tests of Fixed Effects

Num Den

Effect DF DF F Value Pr > F

A 1 106 0.22 0.6409

B 1 106 3.03 0.0849

A\*B 1 106 0.67 0.4152