Uncertainty Analysis in a Field-scale P Loss Model

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Sources of Model Uncertainty

- Model structure error
  - All models are approximations
  - “All models are wrong, some are useful”
- Model parameter error (Generally obtained through calibration)
  - Incorrect optimization targets
  - Inaccurate, incomplete, or unrepresentative calibration data
- Model input error (variables such as rainfall, soil test P)
  - Measurement errors
  - Unrepresentative values

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USDA ARS United States Department Of Agriculture Agricultural Research Service

U.S. Dairy Forage Research Center
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Products & Services

APLE Homepage

Annual Phosphorus Loss Estimator

***APLE** is a spreadsheet model that simulates dissolved and sediment bound phosphorus loss in surface runoff.*

Click here to download spreadsheet for
- Data Entry
- Output Graphs
- Calculations

Click here to view APLE User's Manual Version 2.0, Spring 2011

Click here to view APLE Theoretical Documentation Version 2.0, Spring 2011

For user questions or to report potential errors in calculations, contact the **APLE** creator:

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Objective:

Evaluate model predictions when uncertainties in both model output and measurements are included

How large are the errors?

Do uncertainties help in model evaluation?

APLE input model variables

P Loss Pathway			
DP_{man}	DP_{fert}	DP_{soil}	P_{sed}
Runoff/Precip	Runoff/Precip	Runoff	Soil Loss
Total manure applied	Total P applied	Labile P	Labile P
Percent manure solids	Percent fertilizer incorporation		Soil clay content
Manure TP content			Soil organic matter content
Water extractable P			
Percent manure incorporation			
Mineralization rate			

Assumed Errors in Model Input Variables

Model Variable	Small Uncertainty	Large Uncertainty
Runoff (weir)	± 5%	± 10%
Runoff (direct)	± 1%	± 3%
Erosion	± 2.5%	± 5%
Manure mineralization	± 2.5%	± 5%
P incorporation rates	± 5% (constant)	± 10% (constant)
All other variables	± 5%	± 15%

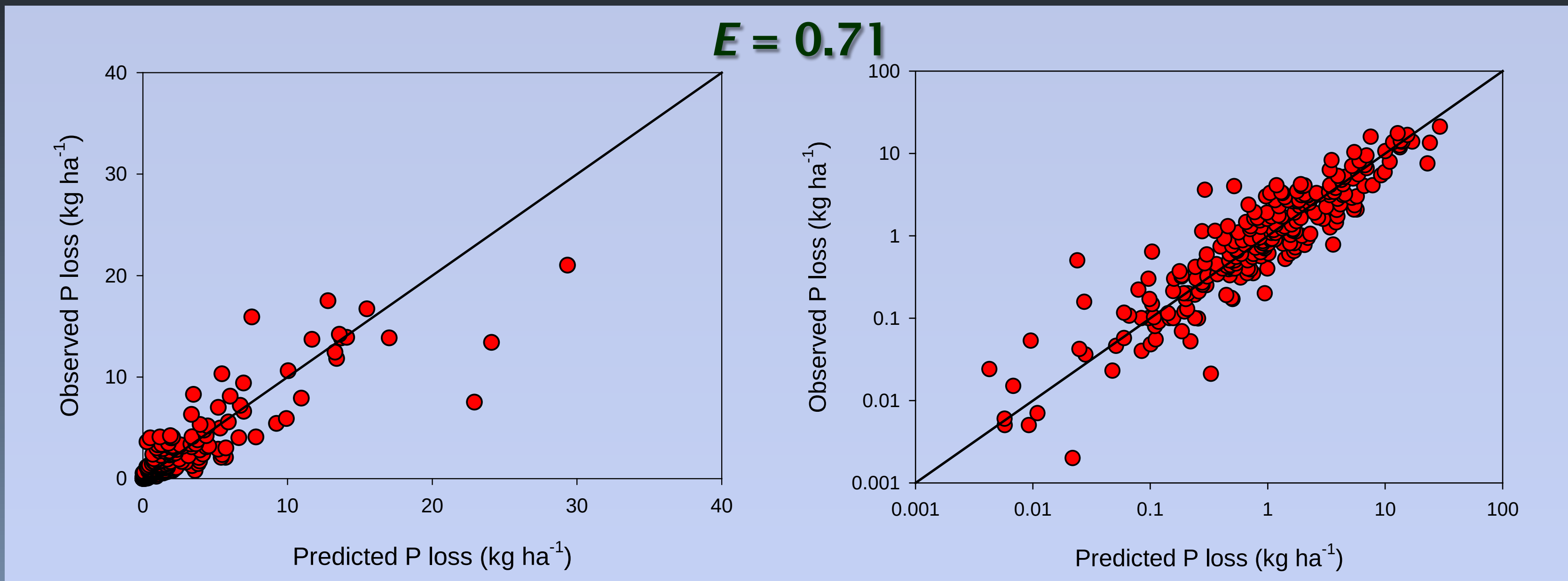
Based on Harmel et al. 2006

Assumed Errors in Measured P Loads

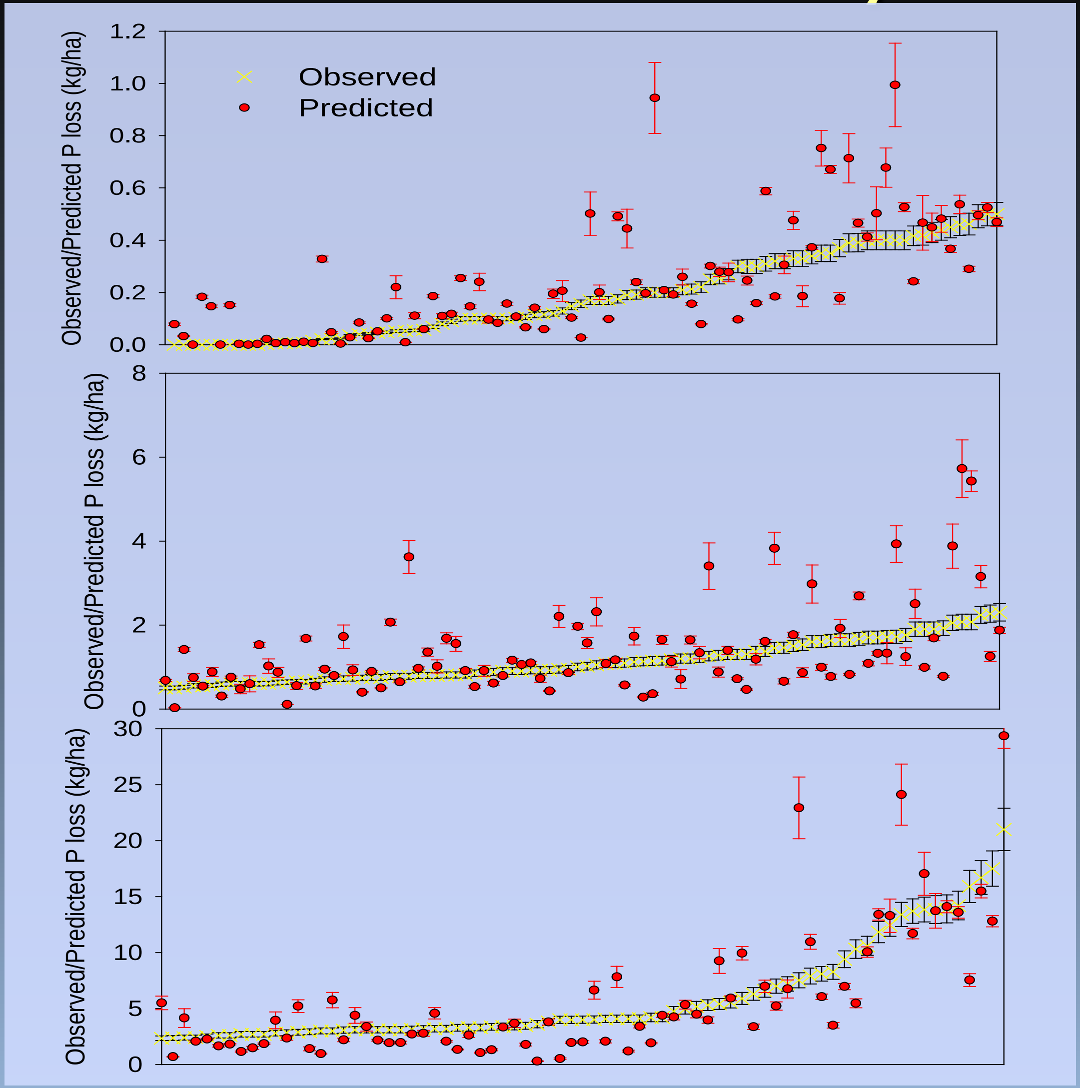
Model Variable	Small Uncertainty	Large Uncertainty
Sample collection	± 5%	± 15%
Sample preservation	± 5%	± 15%
Laboratory analysis	± 5%	± 20%
Contributing area	± 2%	± 5%
Total Error in P Loads	± 9%	± 29%

Based on Harmel et al. 2006

Correlations between predicted and observed P loss



Small Uncertainty

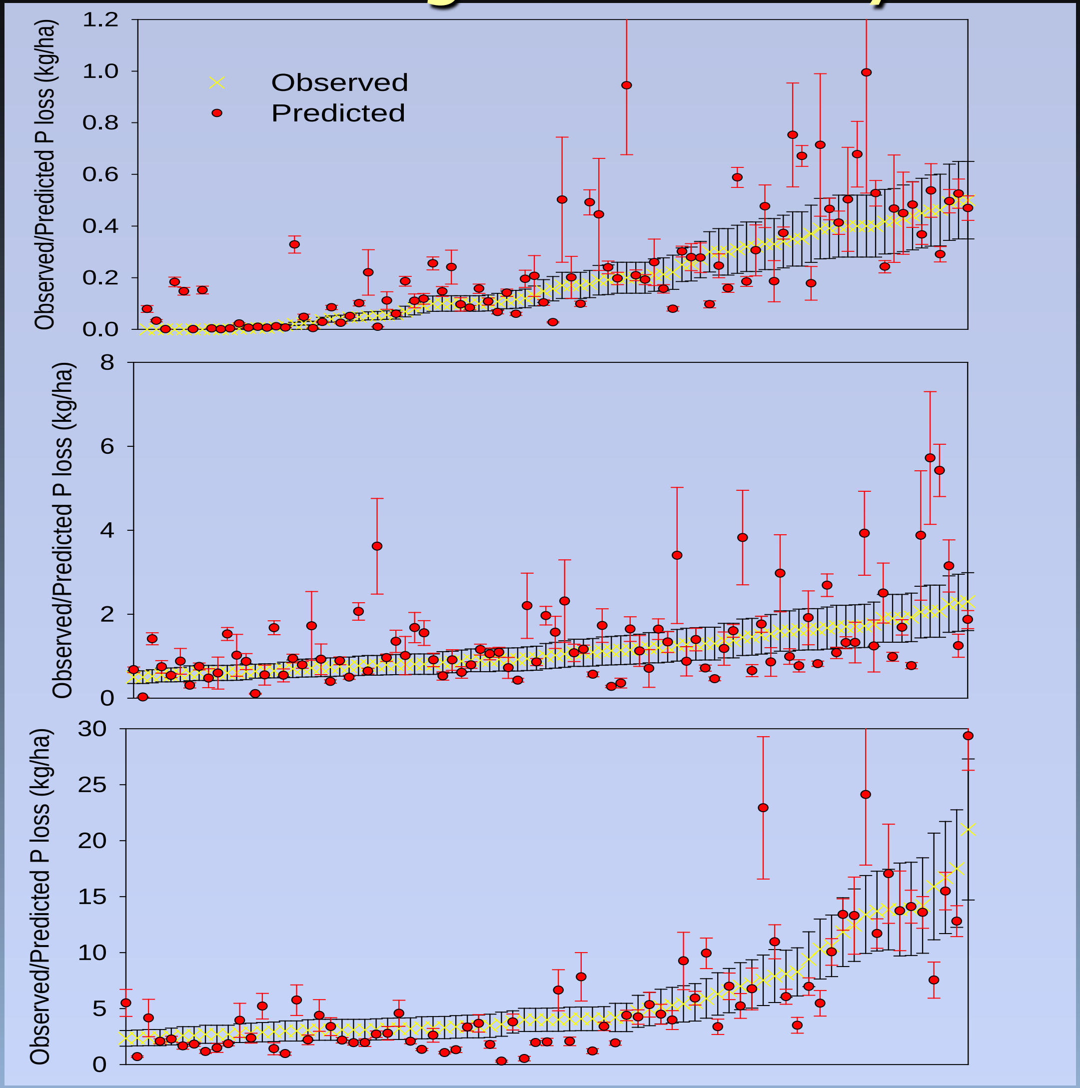


CI range
± 3.5 x10^{-5} to 2.8 kg/ha

CI range
± 2 to 32 %

overlapping CIs
66 out of 255

Large Uncertainty

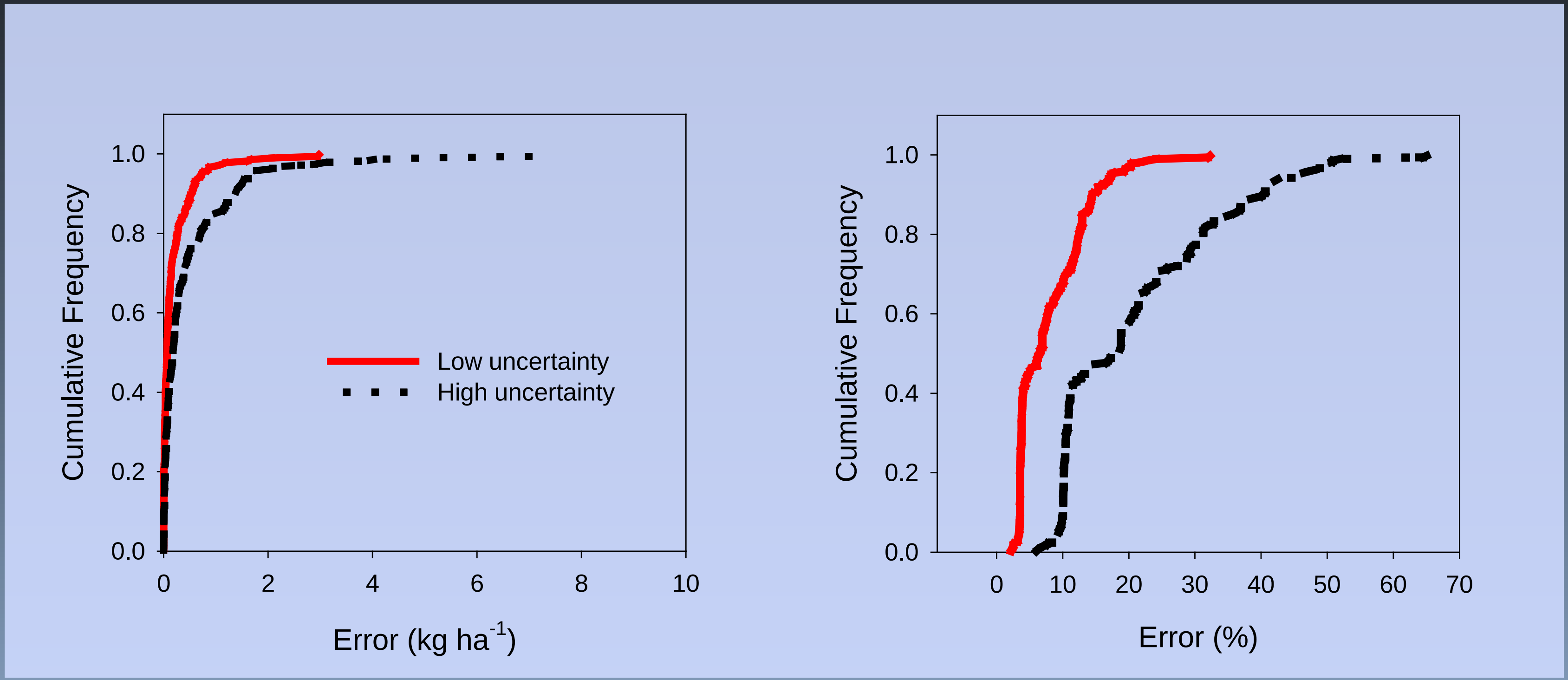


CI range
$\pm$ 9.4 x10^{-5} to 6.4 kg/ha

CI range
$\pm$ 6 to 64 %

overlapping CIs
156 out of 255

Distributions of absolute and relative errors for model predictions



Goodness-of-fit statistics

Statistic	No uncertainty	Small uncertainty	Large uncertainty
E	0.71	0.71	0.72
RMSE, kg P ha^{-1}	1.84	1.84	1.82
MAE, kg P ha^{-1}	0.88	0.87	0.83
MAPE, %	59.1	58.8	57.0
# of overlapping CIs		65	155

Conclusions

- Uncertainties in model predictions are a fact of life
- Ignoring them may do more harm than good
- Uncertainties in model predictions can help us better evaluate our models
- As modelers it is our responsibility to faithfully present the limitations with our model predictions to our audience

Questions?

FOA Method

- Calculates variance in model output as product of input variable variance and sensitivity of model output to changes in that variable

$$\sigma^2(\Theta) = \sum_{i=1}^{k} \left(\frac{\partial \Theta}{\partial I_i} \right)^2 \cdot \sigma^2(I_i)$$

$$\frac{\partial \Theta}{\partial I} \approx \frac{\Theta_{I+\Delta I} - \Theta_{I-\Delta I}}{2\Delta I}$$

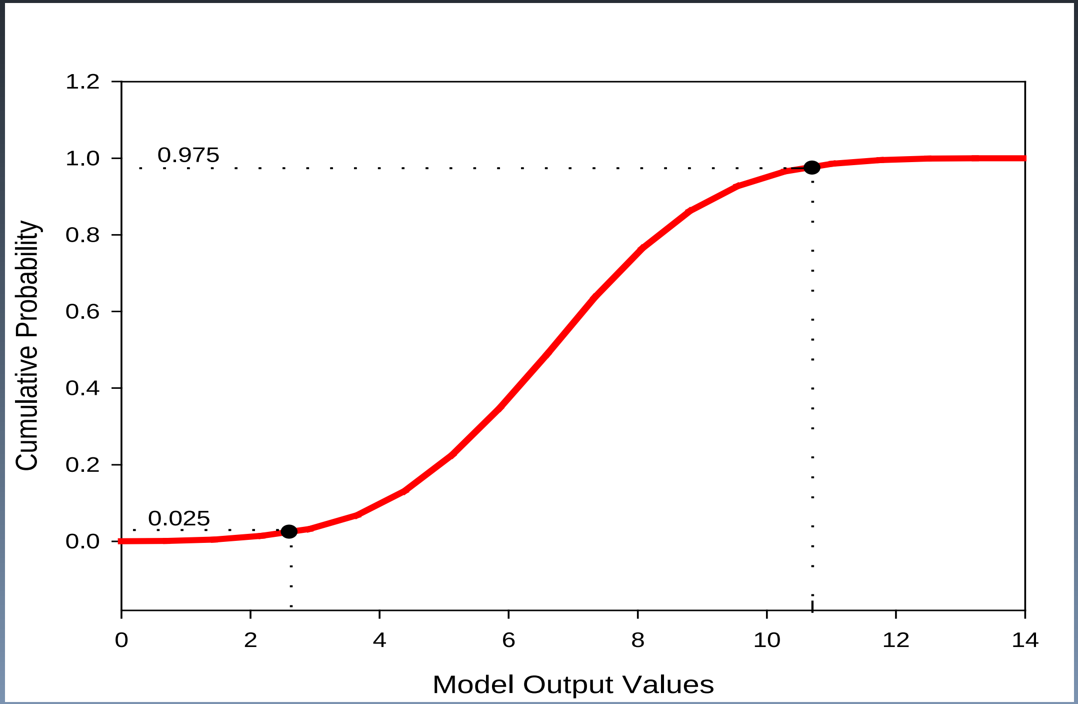
θ is model output

I is model input variable

MCS Method

- Model input variables are selected randomly from a pre-defined distribution (triangular)
- The model is run, and the output is stored.
- The process is repeated numerous times
- Statistical distributions of the output ensemble are used to assess uncertainty in model output.

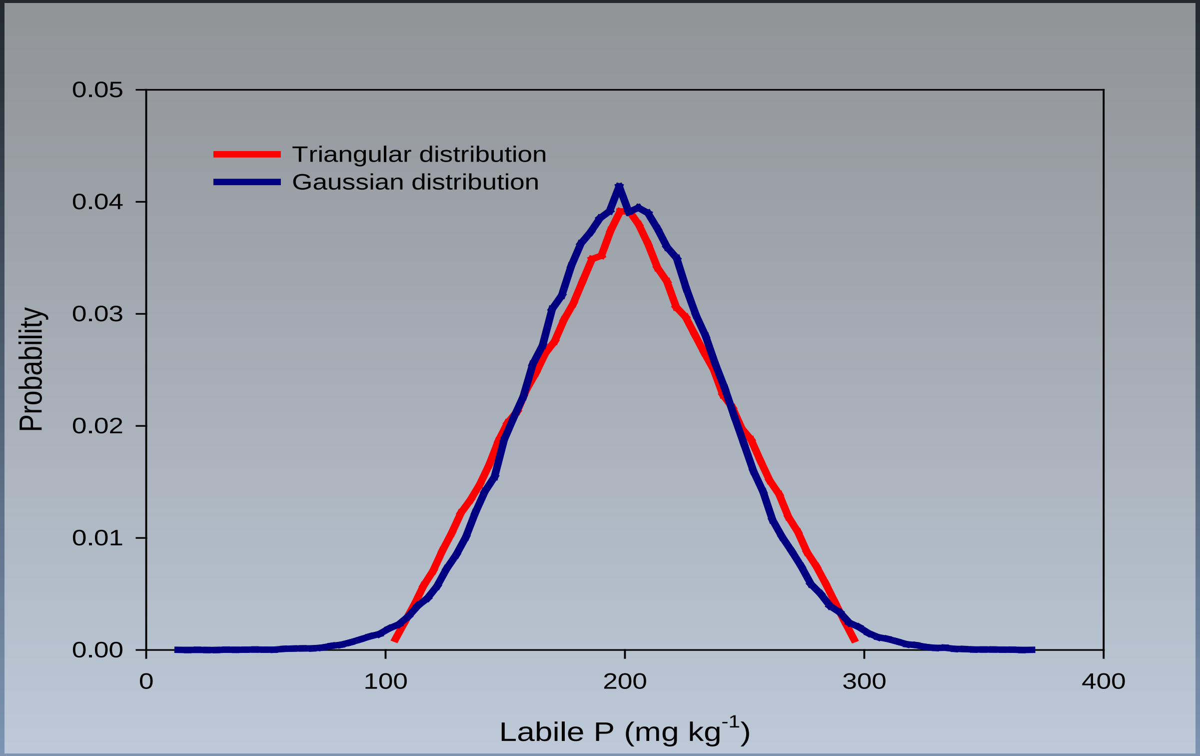
95% confidence intervals for MCS



Methods

- Simulated P loss from two different field conditions:
  - DRP from soil, particulate P loss, DRP from fertilizer
  - DRP from soil, particulate P loss, DRP from manure
- Four error ranges:
  - ± 5%, ± 15%, ± 25%, ±50%
  - Triangular distribution

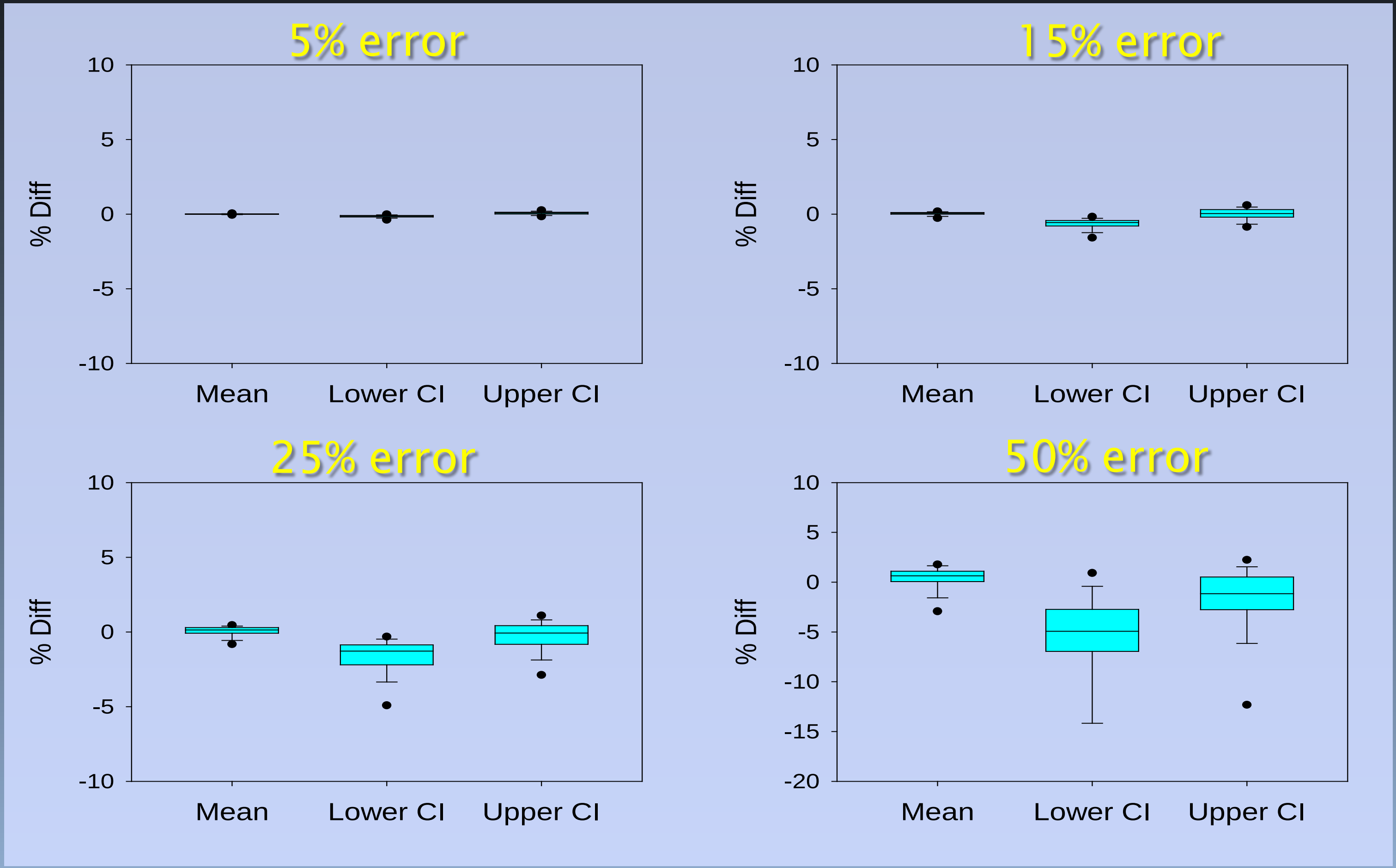
Triangular vs Normal distribution



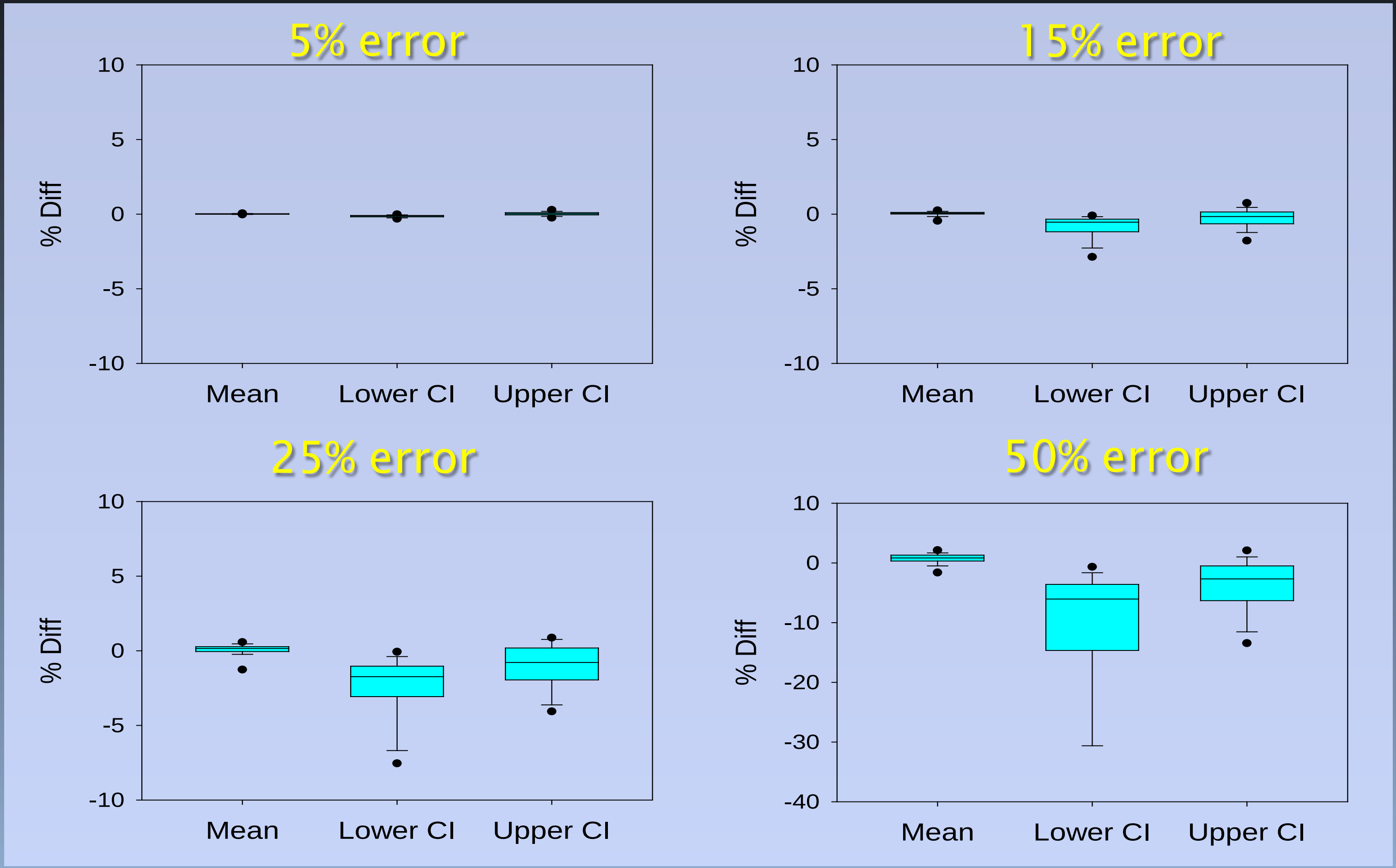
Objective 1:

Compare First-Order Approximation (FOA) method with Monte Carlo Simulation (MCS) Method using APLE

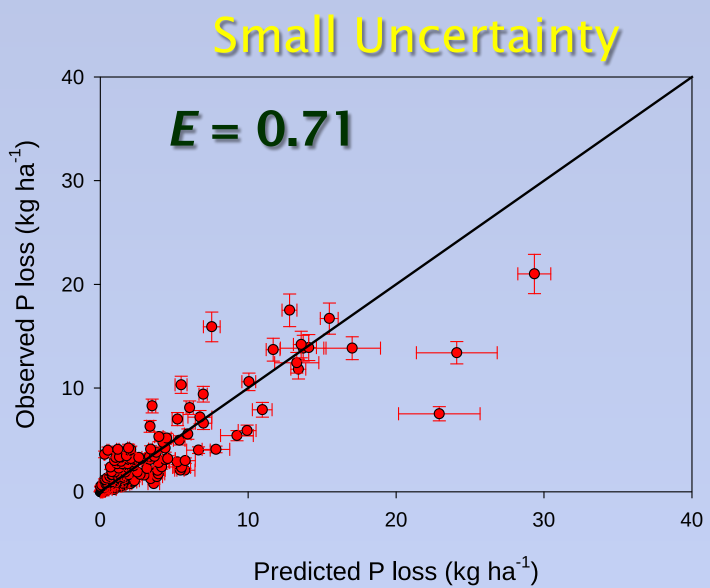
% Differences in CIs between MCS and FOA:
P loss from STP, erosion, and fertilizer P

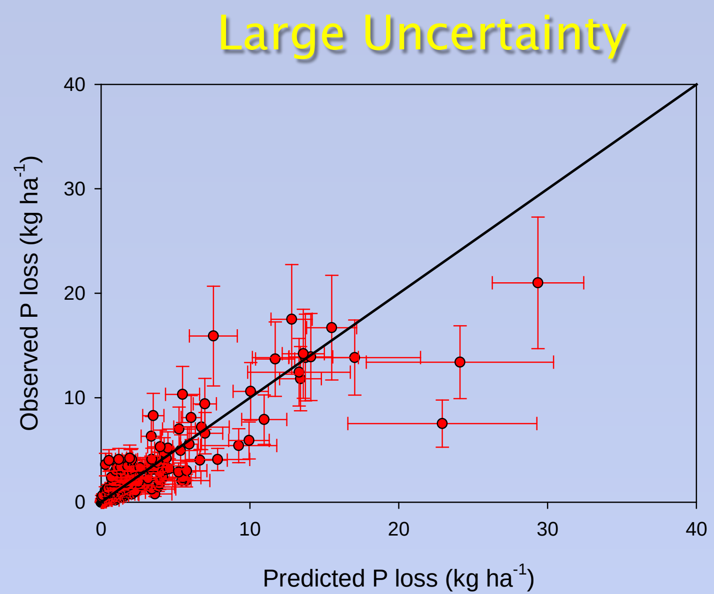


% Differences in CIs between MCS and FOA:
P loss from STP, erosion, and manure P

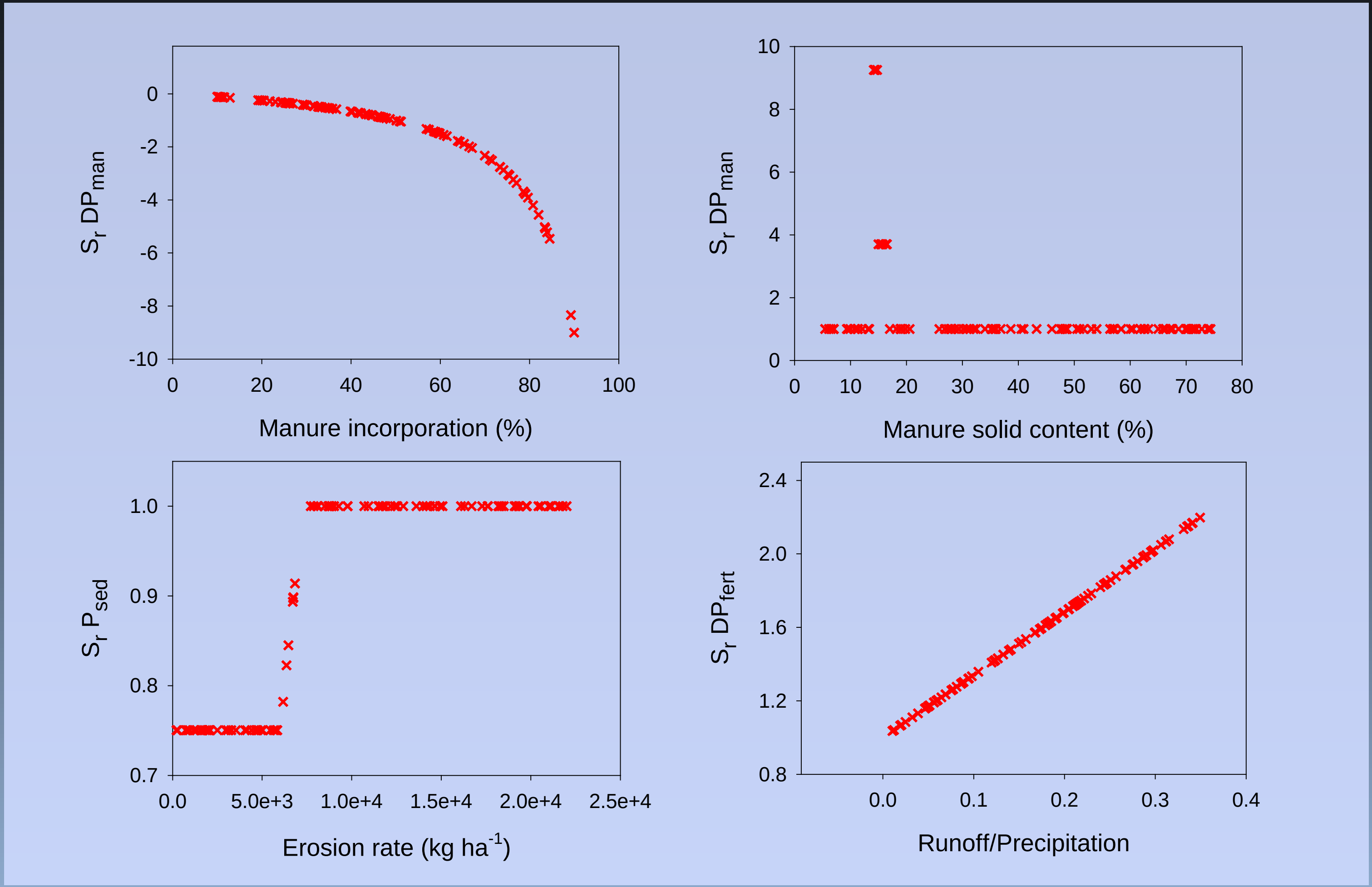


Correlations between predicted and measured P loss





Sensitivity Coefficients



Objectives

1. Conduct sensitivity analysis with all APLE input variables
   - Help identify which input variables require most accurate measurements

2. Evaluate model predictions when uncertainties in both model output and measurements are included

Objective 1:

Sensitivity Analysis of APLE Input Variables

Relative Sensitivity Coefficient

- **Dimensionless parameter that measures how sensitive model output (θ) is to a given change in model input (I)**

$$S_r = \frac{\partial \Theta}{\partial I}\frac{I}{\Theta}$$

$$\frac{\partial \Theta}{\partial I} \approx \frac{\Theta_{I+\Delta I} - \Theta_{I-\Delta I}}{2\Delta I}$$

θ is model output

I is model input variable

Sensitivity Coefficients

Model Variable	Variable Range	S_r mean and range
	DP_{soil}	
Runoff, mm	10 – 500	1.0
Labile soil P, mg kg^{-1}	10 – 400	1.0
	DP_{man}	
RO/PT, %	1 – 35	1.2
Manure application rate, kg ha^{-1}	$2.2 \times 10^2 – 5.6 \times 10^4$	1.0
Solid content of manure, %	5 – 75	1.2 (1.0, 9.3)†
P content of manure, %	0.1 – 5	1.0
WEP content of manure, %	25 – 75	0.76 (0.51, 0.93)
Manure incorporation rate, %	10 – 90	-1.5 (-9.0, -0.11)
	DP_{fert}	
RO/PT, %	1 – 35	1.8 (1.3, 2.2)
Fertilizer application rate, kg ha^{-1}	20 – 300	1.0
Fertilizer incorporation rate, %	10 – 90	-1.5 (-9.0, -0.11)
	P_{sed}	
Erosion rate, kg ha^{-1}	$22 – 5.6 \times 10^4$	0.93 (0.75, 1.0)
Labile P, mg kg^{-1}	10 – 400	0.19 (-0.30, 0.80)
Clay content, %	0.1 – 50	0.19 (0.10, 0.59)
Soil organic matter, %	0.1 – 10	0.46 (0.02, 2.2)

Objectives

1. Test whether the first-order approximation method can provide accurate estimates of confidence intervals for APLE predictions
   - Compare results with Monte Carlo simulations

2. Conduct sensitivity analysis with all APLE input variables
   - Help identify which input variables require most accurate measurements

3. Evaluate model predictions when uncertainties in both model output and measurements are included

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